Diffraction and Decoherence

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1. Introduction. In a famous series of discussions Einstein and Bohr have shed light on several topics in quantum mechanics. One of these is the double slit interference experiment [1]. Quantum mechanics predicts an interference pattern, but only if we have no knowledge of which slit the particle went through. Einstein devised a thought-experiment that would measure through which slit the particle went *and* show the interference pattern. If such an experiment could be performed he hoped that it would show that quantum mechanics is incomplete. Einstein considered a particle beam illuminating a single slit screen placed in front of a double slit screen. Ingeniously he suggested that the recoil of the single slit screen could be measured to determine through which slit the particle would move. Because the recoil would be present regardless if it is measured or not, it appears that such a measurement would not affect the experiment in any way, and the interference pattern would remain. Bohr replied that when we measure the recoil, i.e. the momentum of the screen, accurately enough to determine through which slit the particle went, the uncertainty in our knowledge of the position of the slit is so large that the interference pattern is obscured. In other words, quantum mechanics, through Heisenberg's uncertainty relation, protects itself. In a later treatise Wootters and Zurek analyze this thought experiment quantitatively [2]. This allows relating the probability of going through one slit with the contrast of the interference pattern.

It is interesting to note that in both these discussions the single slit screen is described quantum mechanically. But a single slit screen is a macroscopic object, which appears to be in conflict with its quantum mechanical description. This issue has to our knowledge not been quantitatively analyzed earlier. This issue becomes conceptually more interesting when we realize that the single slit diffraction effectively creates an entangled state,

$$\left|\psi(-k)\right\rangle_{slit}\left|\psi(k)\right\rangle_{particle}+\left|\psi(k)\right\rangle_{slit}\left|\psi(-k)\right\rangle_{particle}.$$

Would not decoherence of the slit state remove the same coherence we need to see interference in the particle state? The main purpose of this paper is to address this question.

Part I.

First we will summarize the main results of Wootter and Zurek's approach and relate their work to the question posed above. We will use the same notation as used by these authors. The system discussed is schematically presented in figure 1. The position of the single slit plate is given by z, the position of the photon at the detection screen is given by ξ, the positions of each of the double slits of labeled with A and B and the observed photon distribution is indicated by $f(\xi)$.

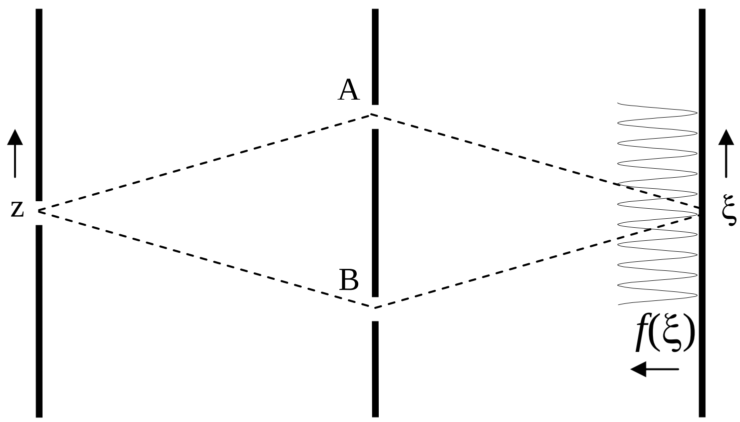

Figure 1. Schematic of Zurek's analysis of the Bohr-Einstein double slit thought experiment.

The wavefunction describing both the slit and photon in position representation is given by

$$\psi(z,\xi)=\frac{1}{\sqrt{2a}\pi^{1/4}}\int e^{-x^2/2a^2}[e^{ik_0(\xi+x)}+e^{-ik_0(\xi+x)}]\delta(z-x)dx\,. \qquad 1.1$$

The momentum of the photon is given by , and the parameter a is a measure of the width of the Gaussian distribution of the slit. If a=0 then we know where the slit is, but we do not know what its momentum is, and the interference pattern has full contrast. Einstein modification of the double slit experiment means we would know something about the momentum of the slit, which, as Bohr pointed out, would smear out the position of the slit, because of Heisenberg's uncertainty principle, and thus cause the interference contrast to disappear. Wootter and Zurek's equation 1.1 allows that statement to be made quantitative. For a given value of a, the intereference pattern can be calculated;

$$f(\xi)=\int|\psi(z,\xi)|^2dz\,, \qquad 1.2$$

by integrating over the unobserved slit position z. Specifically,

$$\begin{aligned} f(\xi)&=\int|\psi(z,\xi)|^2dz=\\ &=\int\frac{1}{2a\sqrt{\pi}}e^{-z^2/a^2}(e^{ik_0(\xi+z)}+e^{-ik_0(\xi+z)})(e^{-ik_0(\xi+z)}+e^{ik_0(\xi+z)})=\\ &=\int\frac{1}{a\sqrt{\pi}}e^{-z^2/a^2}(1+\cos(2k_0(\xi+z))=\\ &=1+e^{-a^2k_0^2}\cos(2k_0\xi) \end{aligned} \qquad 1.3$$

In the first step the intergration over the delta-function, $\delta(z-x)$, has been performed, while in the last step the integral, $\int \frac{1}{a\sqrt{\pi}} e^{-z^2/a^2}\cos(2k_0 z) = e^{-a^2k_0^2}$, has been used. In the limit of a->0 the position of the slit is exactly know, and the contrast is complete. We now turn our attention to the wavefunction of the slit and the photon in the momentum representation. Wootters and Zurek state that it can be verified that this is "equal" to the position distribution, and give

$$\psi(k_0,\xi) = \frac{a}{2\pi^{1/4}} \int D^{1/2}(\kappa)[p_A{}^{1/2}(\kappa) e^{ik_0\xi} + p_B{}^{1/2}(\kappa) e^{-ik_0\xi}] e^{i\kappa z} d\kappa \ . \qquad 1.4$$

In this expression, κ, is slit momentum, while the functions D and $p_{A,B}$ are given by,

$$\begin{aligned} D(\kappa) &= \frac{a}{2\sqrt{\pi}}\left\{ e^{-a^2(\kappa+k_0)^2} + e^{-a^2(\kappa-k_0)^2} \right\} \\ p_A\big/ p_B &= e^{-a^2(\kappa+k_0)^2} \big/ e^{-a^2(\kappa-k_0)^2} \ ; \ p_A + p_B = 1 \end{aligned} \ . \qquad 1.5$$

The functions $p_{A,B}$ give the probability of going through slit A or B, respectively, and D normalize the wavefunction. The limit $a \to \infty$ justifies the statement in the introduction that diffraction essentially creates and entangled state. In this limit the probabilities act as delta-functions, $\delta(\kappa+k_0), \delta(\kappa-k_0)$. After integration the wavefunction becomes

$$\begin{aligned} \lim_{a\to\infty} \psi(k_0,\xi) &= \tfrac{1}{2}\sqrt{2}(e^{ik_0\xi}e^{-ik_0 z} + e^{-ik_0\xi}e^{ik_0 z}) = \\ &= |\psi(-k_0)\rangle_{slit} |\psi(k_0)\rangle_{photon} + |\psi(k_0)\rangle_{photon} |\psi(-k_0)\rangle_{slit} \end{aligned} \ . \qquad 1.6$$

We can now proceed and calculate the probability to find the photon on position ξ on the detection screen,

$$\begin{aligned} f(\xi) &= \int \lim_{a\to\infty} |\psi(z,\xi)|^2 dz = \\ &= \lim_{a\to\infty} \frac{1}{(2a)^2} \int_{-a}^{a} 1 + \cos(2k_0(\xi - z))dz = 1/2a \end{aligned} \ , \qquad 1.7$$

In words, this states that, if a is made very large then the momentum of the slit is known, and at the same time the interference pattern is known. Combining this with the earlier statement that for very small a the position of the slit is know, while at the same time the interference pattern has full contrast, it can be recognized that this result can be reached by using Bohr's argumentation using the Heisenberg uncertainty relation. Specifically, if the momentum of the slit is measured exactly the position is completely unknown and incoherently averaging over this position blurs the interference pattern completely. Wootters and Zurek not only justified this reasoning, but also allow give quantative expressions when not considering the extreme cases. The probability to go through one of the holes and the interference contrast can be calculated for arbitrary values of a.

A final observation should be made, WZ point out that the photon states need not be orthogonal. This will turn out to be a central point in our discussion.

Part II.

Now we return to our original question and we allow the quantum mechanical state of the slit to decohere. For a realistic macroscopic single slit screen this coherence would occur extremely rapidly.

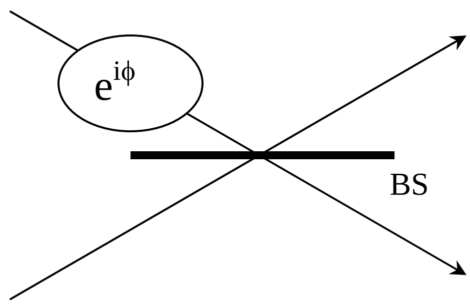

Figure 2. A beam splitter and a phase shifter. This toy system will be used throughout the paper as an example.

If such decoherence does not change Bohr's reasoning and WZ's justification, then we have substantiated that the macroscopic single slit screen can be described by a quantum mechanical wavefunction. Because experimentally interference is observed, regardless of the fact that the single slit decoheres rapidly, one should of course anticipate that such a substantiation can indeed be provided.

To investigate this problem we will use as a toy system a beam splitter (BS) combined with a phase shifter (fig 2). The beam splitter is thought to play the role of the single slit. Because for the single slit only two momentum states are important (the ones that make it to each of the slits in the double slit screen) a beam splitter appears to be a convenient substitute for the single slit. The basic concept that recoil takes place is shared between single slit diffraction and beam splitter reflection. For the double slit interferometer fringes can be seen as a function of position, while for the beam splitter an additional phase-shifter is added to allow the observation of fringes. In the section 2 we will give the usual description of two coherent beams passing through a beam splitter, while in section 3 we will add the quantum mechanical description of the beam splitter itself. In section four decoherence of the beam splitter will be added, while in section five the interfering particle beams will be allowed to decohere.

2. A beam splitter.

a. Wave function approach. Assume the input state is given by

$$|\psi\rangle_1 = c_{-k}|-k\rangle + c_k|k\rangle\,, \qquad 2.1$$

where –k and k correspond to the two momentum state of the incoming coherent beams (fig 2.). After passing through the phase shifter the state will be given by

$$|\psi\rangle_2 = c_{-k}e^{i\varphi}|-k\rangle + c_k|k\rangle\,. \qquad 2.2$$

Beams that reflect from the beam splitter will pick up the usual $e^{i\pi/2}$ phase shift, so that after the beam splitter the output state is described by

$$|\psi\rangle_3 = (\tfrac{\sqrt{2}}{2}c_{-k}e^{i\varphi} + \tfrac{\sqrt{2}}{2}e^{i\pi/2}c_k)|-k\rangle + (\tfrac{\sqrt{2}}{2}c_{-k}e^{i\varphi}e^{i\pi/2} + \tfrac{\sqrt{2}}{2}c_k)|k\rangle\,. \qquad 2.3$$

It is convenient to express the action of the beam splitter in terms of an operator, A_{BS},

$$A_{BS} = \tfrac{\sqrt{2}}{2}\begin{pmatrix}1 & i\\ i & 1\end{pmatrix}, \qquad 2.4$$

such that

$$A_{BS}|\psi_2\rangle = |\psi_3\rangle\,. \qquad 2.5$$

Alternatively, we can write

$$A_{BS} = \tfrac{\sqrt{2}}{2}(1 + i(p^\dagger + p))\,, \qquad 2.6$$

where $p^\dagger$ and p are operators that raise and lower the particle's momentum. In matrix form they take the form

$$\begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix} \text{ and } \begin{pmatrix}0 & 0\\ 1 & 0\end{pmatrix}. \qquad 2.7$$

This expression will be helpful when we will later generalize this expression. The probabilities to observe a particle leaving the beam splitter with momentum k or –k can now be given directly by squaring the amplitudes in eq 2.3. For example, when $c_{-k} = c_k = \sqrt{2}/2$ these probabilities are $\tfrac{1}{2} + \tfrac{1}{2}\sin(\varphi)$ and $\tfrac{1}{2} - \tfrac{1}{2}\sin(\varphi)$, respectively.

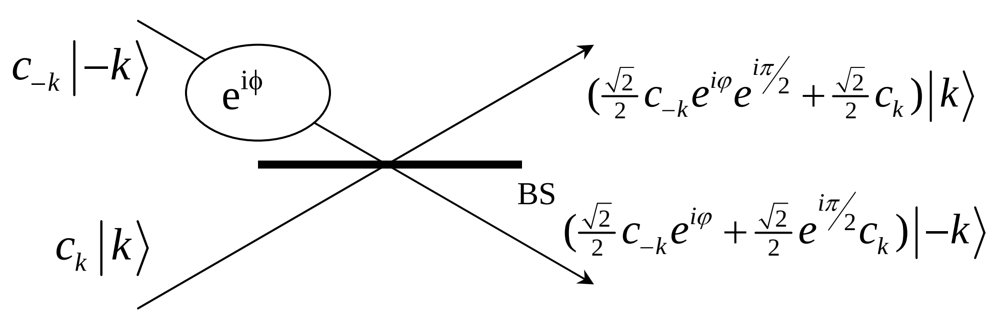

Figure 3. The effect of a beam splitter. The incoming and outgoing momentum particle states are indicated.

b. Density matrix approach. It is useful to also cast the above statements in the form of density matrices because in later section we will add decoherence. The density matrix describing the input state is

$$\rho_1 = \begin{pmatrix} c_{-k}^* c_{-k} & c_{-k}^* c_k \\ c_k^* c_{-k} & c_k^* c_k \end{pmatrix} = \begin{pmatrix} \rho_{-k,-k} & \rho_{-k,k} \\ \rho_{k,-k} & \rho_{k,k} \end{pmatrix}. \qquad 2.8$$

The phase shifter will add a term $e^{\pm i\varphi}$ for each –k term:

$$\rho_2 = \begin{pmatrix} e^{-i\varphi} c_{-k}^* e^{i\varphi} c_{-k} & e^{-i\varphi} c_{-k}^* c_k \\ c_k^* e^{i\varphi} c_{-k} & c_k^* c_k \end{pmatrix} = \begin{pmatrix} \rho_{-k,-k} & e^{-i\varphi} \rho_{-k,k} \\ e^{i\varphi} \rho_{k,-k} & \rho_{k,k} \end{pmatrix}. \qquad 2.9$$

The effect of the beam splitter on the density matrix can either be obtained by using eq. 2.3 or by letting the operator A_{BS} work on ρ_2. The result of this operator is

$$\rho_3 = A^\dagger \rho_2 A = \frac{1}{2} \begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix} \begin{pmatrix} \rho_{-k,-k} & \rho_{-k,k} e^{-i\varphi} \\ \rho_{k,-k} e^{i\varphi} & \rho_{k,k} \end{pmatrix} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}. \qquad 2.10$$

To get the correct probabilities we can now directly read the top left an right bottom element of the density matrix ρ_3.

3. A quantum beam splitter.

a. Wave function approach. Now we would like to add the quantum mechanical description of the motional state of the beam splitter itself. Just as in the earlier work [1,2] we will use momentum conservation to allow us to write these states. The input state is given by

$$|\psi\rangle_1 = |\psi\rangle_1^b \otimes |\psi\rangle_1^p = |0\rangle^b \otimes (c_{-k} |-k\rangle^p + c_k |k\rangle^p), \qquad 3.1$$

here the superscripts b and p indicate the beam splitter and particle, respectively. We could at this point also choose a distribution of beam splitter momentum states and follow the same reasoning as given. After passing the phase shifter the system state becomes

$$|\psi\rangle_2 = |\psi\rangle_2^b \otimes |\psi\rangle_2^p = |0\rangle^b \otimes (c_{-k} e^{i\varphi} |-k\rangle^p + c_k |k\rangle^p) = c_{-k} e^{i\varphi} |0,-k\rangle + c_k |0,k\rangle. \qquad 3.2$$

After passing the beam splitter the momentum states of the beam splitter and particle become entangled,

$$\left|\psi\right\rangle_3 = \tfrac{\sqrt{2}}{2} c_{-k} e^{i\varphi} \left|0,-k\right\rangle + \tfrac{\sqrt{2}}{2} e^{i\pi/2} c_k \left|2k,-k\right\rangle + \tfrac{\sqrt{2}}{2} c_{-k} e^{i\varphi} e^{i\pi/2} \left|-2k,k\right\rangle + \tfrac{\sqrt{2}}{2} c_k \left|0,k\right\rangle . \qquad 3.3$$

This is the point where momentum conservation is used. To obtain the same probability as before, i.e., when measuring only the particle in the –k or k state, we will have to sum over the unobserved beam splitter state, k_b, which indeed gives the correct probabilities;

$$P_{\pm k} = \left| \sum_{k_b} \left\langle \pm k, k_b \middle| \psi \right\rangle_3 \right|^2 = \tfrac{1}{2} \pm \tfrac{1}{2} \sin\varphi , \qquad 3.4$$

where the last equality holds for the example that $c_{-k} = c_k = \sqrt{2}/2$.

b. Density matrix approach. Again we will cast the above statements in terms of the density matrix. The total system is described by the tensor product of the beam splitter density matrix and the particle density matrix,

$$\rho_s = \rho_{(beamsplitter)} \otimes \rho_{(particle)} . \qquad 3.5$$

The density matrix for the particle is the same as in section 2b, while the density matrix for the beam splitter is a 3 by 3 matrix, because the beam splitter can have momenta -2k, 0, or 2k. In particular we can write

$$\rho_1 = \rho_1^b \otimes \rho_1^p = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \otimes \begin{pmatrix} \rho_{-k,-k} & \rho_{-k,k} \\ \rho_{k,-k} & \rho_{k,k} \end{pmatrix} \text{ and} \qquad 3.6$$

$$\rho_2 = \rho_2^b \otimes \rho_2^p = \rho_1^b \otimes \rho_2^p , \qquad 3.7$$

where ρ_2^p is given by equation 2.9. As before this is where the effect of the phase shifter enters. After the particle passes the beam splitter we can either construct the density matrix from the wave function (eq. 3.3) or use an operator that describes the appropriate momentum exchange. This operator is

$$A_{BS} = \tfrac{\sqrt{2}}{2} (1 + i(b \otimes p^\dagger + b^\dagger \otimes p)) , \qquad 3.8$$

where $b^\dagger$ and b are the raising and lowering operator for the beam splitter's momentum. The first term indicates that, if the particle does not change its momentum neither does the beam splitter and in matrix from it can be read as the tensor product of the identity matrix of rank 3 with the identity matrix of rank 2. The second and third term indicate that if the particle change momentum then the beam splitter will change momentum in the opposite way. In matrix form they are

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \text{ and } \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}. \qquad 3.9$$

Finally, we can find the density matrix of the beam splitter and particle:

$$\rho_3 = A_{BS}^{\dagger} \rho_2 A_{BS} . \qquad 3.10$$

Explicitly this matrix takes the form for the example $c_{-k} = c_k = \sqrt{2}/2$,

$$\rho_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{4} & -\frac{1}{4} i e^{i\varphi} & -\frac{1}{4} i & \frac{1}{4} e^{i\varphi} & 0 \\ 0 & \frac{1}{4} i e^{-i\varphi} & \frac{1}{4} & \frac{1}{4} e^{-i\varphi} & \frac{1}{4} i & 0 \\ 0 & \frac{1}{4} i & \frac{1}{4} e^{i\varphi} & \frac{1}{4} & \frac{1}{4} i e^{i\varphi} & 0 \\ 0 & \frac{1}{4} e^{-i\varphi} & -\frac{1}{4} i & -\frac{1}{4} i e^{-i\varphi} & \frac{1}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \qquad 3.11$$

and just as eq.3.3 it is describing an entangled state. Tracing over the beam splitter quantum numbers gives

$$\rho_{3,particle} = \begin{pmatrix} \frac{1}{2} & \frac{1}{4} e^{i\varphi} \\ \frac{1}{4} e^{-i\varphi} & \frac{1}{2} \end{pmatrix}. \qquad 3.12$$

Tracing over the particle quantum numbers gives, (for $\varphi = 0$),

$$\rho_{3,bs} = \begin{pmatrix} \frac{1}{4} & -\frac{1}{4} i & 0 \\ \frac{1}{4} i & \frac{1}{2} & \frac{1}{4} i \\ 0 & -\frac{1}{4} i & \frac{1}{4} \end{pmatrix}. \qquad 3.13$$

It is perhaps interesting to note that both the particle and the beamsplitter are partially coherent, and the particle's coherence is not complete as in section 2b.
Also note that the probability to find the particle in one of its states is always ½ and does not depend on φ . This appears incorrect. A way to calculate the correct answer is

$$P(\pm k_{particle}) = \left| \sum_{k_b, k_b^{'}} \left\langle \pm k_p, k_b \left| \rho_3 \right| k_b^{'}, \pm k_p \right\rangle \right|^2 . \qquad 3.14$$

But incorrect is

$$P(\pm k_{particle}) = Tr_{k_b}\rho_{3,\pm k,\pm k} = \left|\sum_{k_b}\left\langle \pm k_p, k_b \left|\rho_3\right| k_b, \pm k_p\right\rangle\right|^2 , \qquad 3.15$$

which gives you the diagonal elements of 3.12. (How can I justify this or write that as a trace in a more recognizable form????? And how do I understand that 3.15 does not give the correct result). For the next section we will use eq. 3.14, because it seems to work.

4. Decoherence of the quantum beam splitter. To describe decoherence in a system the optical Bloch equations can be used,

$$\frac{\partial\rho}{\partial t} = i\left[H, \rho\right] + \Gamma_\rho . \qquad 4.1$$

Because the action of a beam splitter on a quantum mechanical state has already been described, we only need to apply this equation after the particle passed the beam splitter. In that case H=0 and we only need to consider the decoherence. Different decoherence processes can be described. One could follow Bloch's approach and use two separate decay rates; one for polarization and one for coherence. Another possibility could be a process that can couple the two momentum states symmetrically could be described by:

$$\Gamma_\rho = -\Gamma\begin{pmatrix} \rho_{k,k} - \rho_{-k,-k} & \rho_{-k,k} \\ \rho_{k,-k} & \rho_{-k,-k} - \rho_{k,k} \end{pmatrix}. \qquad 4.2$$

Alternatively, a process that only affects the coherences and not the populations could be described by:

$$\Gamma_\rho = -\Gamma\begin{pmatrix} 0 & \rho_{-k,k} \\ \rho_{k,-k} & 0 \end{pmatrix}. \qquad 4.3$$

We choose here the last, because it puts the focus on the coherences which are required to observe the interferences. Recall that our main objective is to show that the beam splitter decoherence does not effect the coherences that are responsible for the particle's interference pattern. For the total system including the three momentum states of the beam splitter, the decoherence matrix can be written as

$$\Gamma = \Gamma_{\rho(beamsplitter)} \otimes \mathrm{I}_{(particle)} , \qquad 4.4$$

where I stand for a 2 by 2 identity matrix. The total system itself is described by the tensor product of the beam splitter density matrix and the particle density matrix,

$$\rho_s = \rho_{(beamsplitter)} \otimes \rho_{(particle)} . \qquad 4.5$$

After the particle has passed the phase shifter and the beam splitter the density matrix of our system is once again described by eq.3.11. Now we can let the decoherence act. The elements in the density matrix that are responsible for the particle's interference (through eq. 3.14 are indicated in a larger size.

$$\rho_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{4} & -\frac{1}{4}ie^{i\varphi} & -\frac{1}{4}i & \frac{1}{4}e^{i\varphi} & 0 \\ 0 & \frac{1}{4}ie^{-i\varphi} & \frac{1}{4} & \frac{1}{4}e^{-i\varphi} & \frac{1}{4}i & 0 \\ 0 & \frac{1}{4}i & \frac{1}{4}e^{i\varphi} & \frac{1}{4} & \frac{1}{4}ie^{i\varphi} & 0 \\ 0 & \frac{1}{4}e^{-i\varphi} & -\frac{1}{4}i & -\frac{1}{4}ie^{-i\varphi} & \frac{1}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \qquad 4.6$$

So we are particularly interested in the explicitly form of the decoherence matrix, eq.4.4, which is

$$\begin{pmatrix} & & -\Gamma\rho_{2k,-k;-2k,k} & & & \\ & & & -\Gamma\rho_{-2k,k;0,-k} & & \\ -\Gamma\rho_{-2k,-k;2k,-k} & & & & -\Gamma\rho_{0,k;2k,-k} & \\ & -\Gamma\rho_{0,-k;-2k,k} & & 0 & 0 & -\Gamma\rho_{2k,k;-2k,k} \\ & & -\Gamma\rho_{2k,-k;0,k} & 0 & 0 & \\ & & & -\Gamma\rho_{-2k,k;2k,k} & & \end{pmatrix}.$$

It is now clear that elements in the density matrix that are responsible for interference correspond the zero-valued elements in the decoherence matrix, and we have obtained our goal, which was to justify the use of a quantum mechanical description of the macroscopic beam splitter.

(Now show that this result is true for the most general decoherence matrix)

5. Decoherence of the interfering particle beams.

For consistency, we should also be able to add decoherence to the particle and find that the interference pattern disappears. There are two cases to consider; a) the beam splitter does not decohere or b) the beam splitter decoheres.
a) In this case the explicit form of the decoherence matrix is given by

$$\Gamma = \mathrm{I}_{(beamsplitter)} \otimes \Gamma_{\rho(particle)} =$$

$$
=\begin{pmatrix}
 & -\Gamma\rho_{0,-k;-2k,-k} & & & & \\
-\Gamma\rho_{-2k,-k;0,-k} & & & & & \\
 & & & -\Gamma\rho_{-2k,k;2k,-k} & & \\
 & & -\Gamma\rho_{2k,-k;-2k,k} & 0 & 0 & \\
 & & & 0 & 0 & -\Gamma\rho_{2k,k;0,k} \\
 & & & & -\Gamma\rho_{0,k;2k,k} &
\end{pmatrix}.
$$

By inspection it immediately follows that the interference pattern should still be present. This we find a surprising result. However, it does not appear to be in conflict with experiment, because the decoherence of a macroscopic object can not be turned off.

b) In the case that both the beam splitter and particle decohere the decoherence matrix is given by

$$
\Gamma = \Gamma_{\rho(beamsplitter)} \otimes \Gamma_{\rho(particle)} =
$$

$$
=\begin{pmatrix}
 & & & -\gamma\rho_{-2k,k;-2k,-k} & & \\
 & & -\gamma\rho_{0,-k;-2k,-k} & & & \\
 & -\gamma\rho_{0,-k;-2k,-k} & & & & -\gamma\rho_{0,-k;-2k,-k} \\
-\gamma\rho_{0,-k;-2k,-k} & & & 0 & -\gamma\rho_{0,-k;-2k,-k} & \\
 & & & -\gamma\rho_{-2k,k;0,k} & 0 & \\
 & & -\gamma\rho_{0,-k;-2k,-k} & & &
\end{pmatrix}
$$

where $\gamma = -\Gamma_{bs}\Gamma_{p}$.

[1] Wheeler and Zurek, Quantum Theory and Measurement p.9-49, 1983

[2] Wootters K. W., and Zurek W. H., Phys. Rev. D19, 473-84 (1979)