When Covariational Reasoning Does Not "Work": Applying Coordination Class Theory to Model Students' Reasoning Related to the Varied Population Schema and Distribution Graphs

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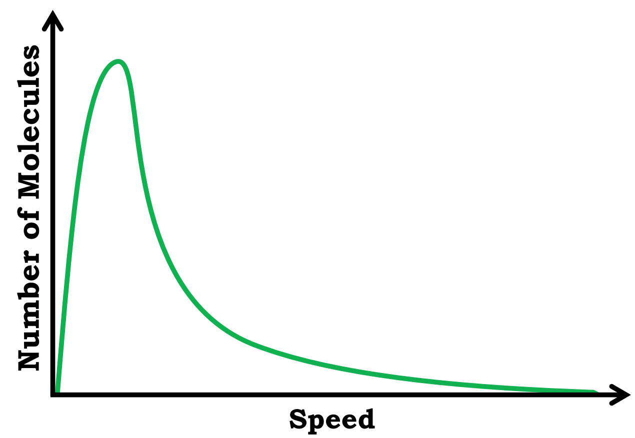
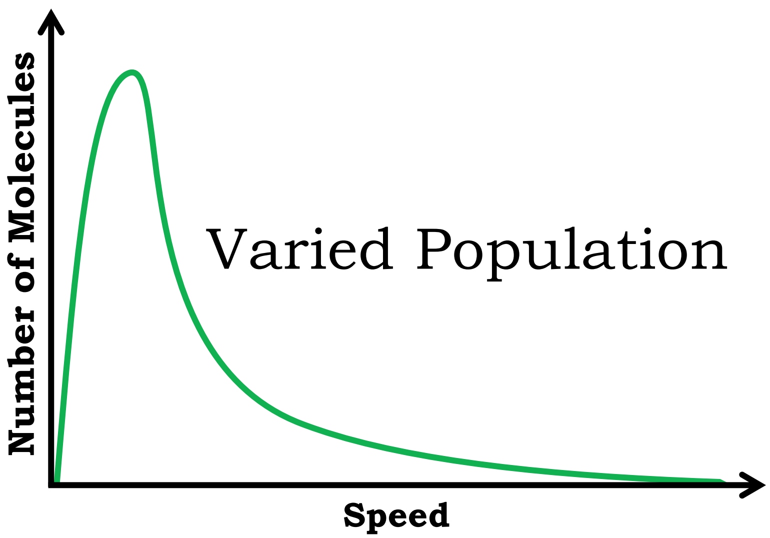
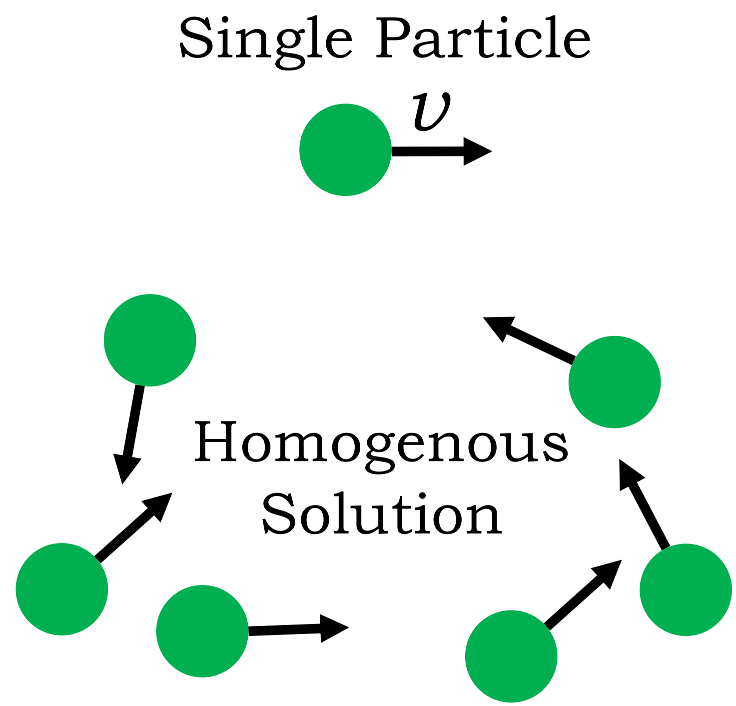


The idea that particles have a distribution of kinetic energies has been described as a critical disciplinary concept at the undergraduate level

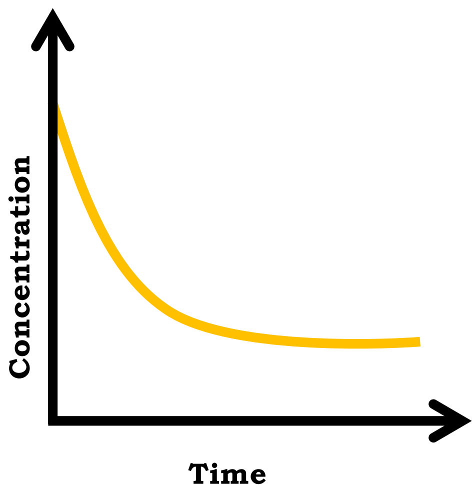


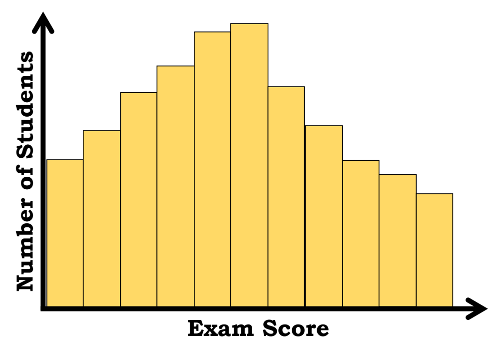
vs.



Holme, Luxford, & Murphy, 2015; Talanquer, 2015

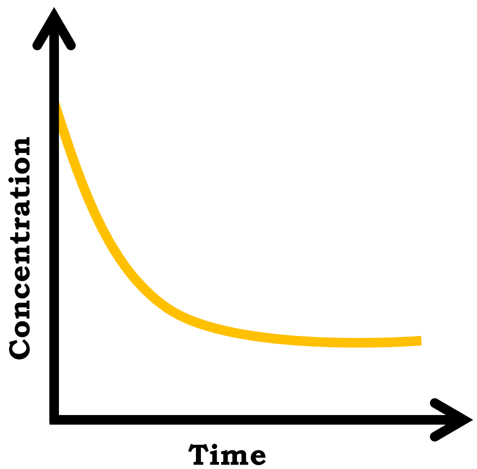
Some graphs can be read using covariational reasoning

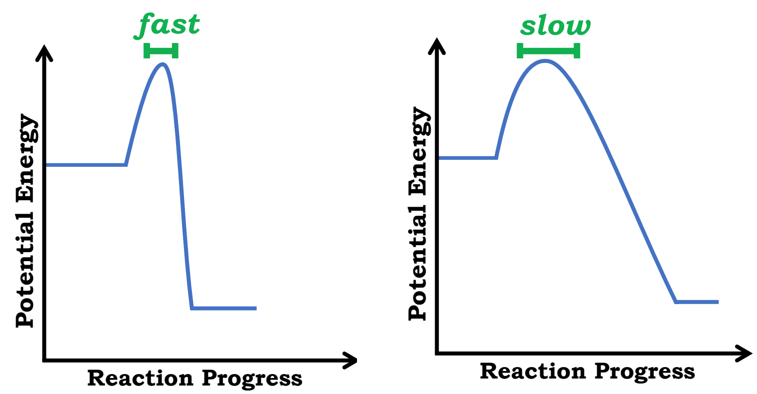




View x-axis as time
(delMas, Garfield, & Ooms, 2005)

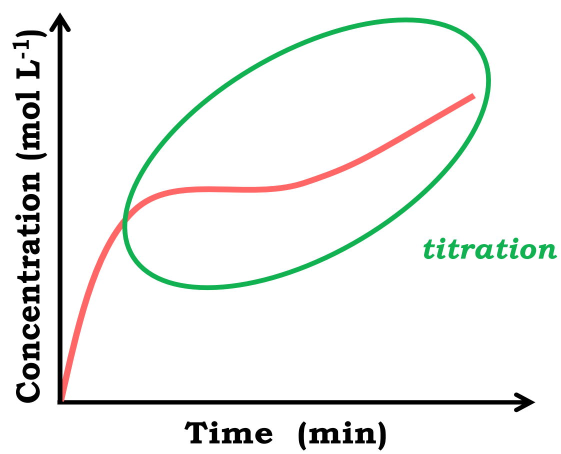
Some graphs can be read using covariational reasoning. Others, less so.





Students may impose time on x-axis (Popova & Bretz, 2018)

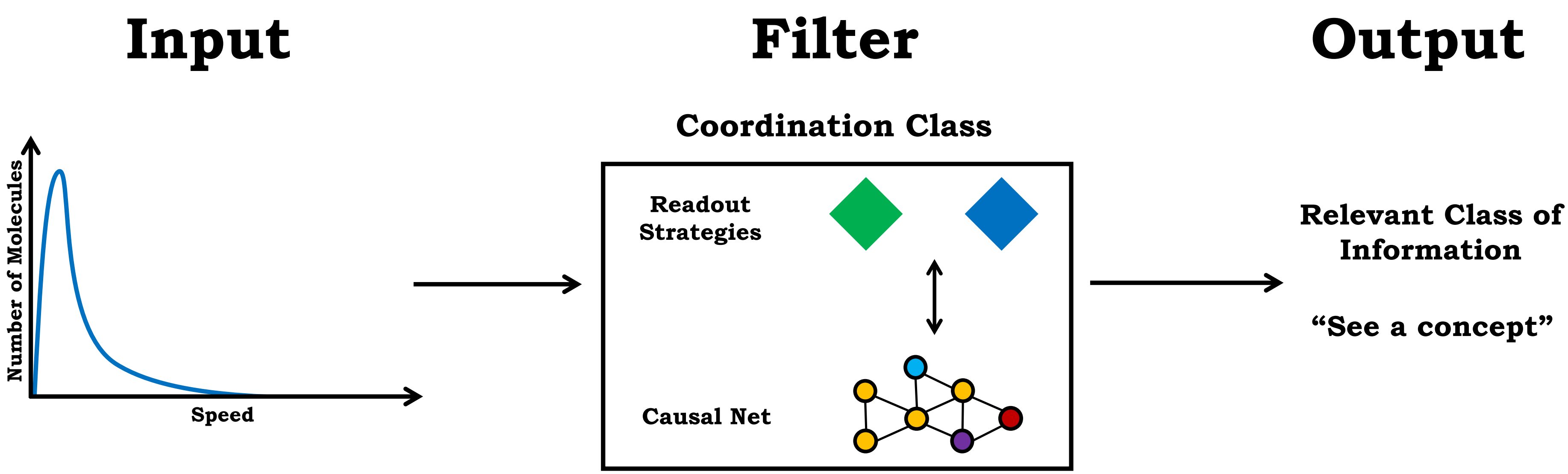
Students may have intuitive ideas associated with graphical patterns



(Rodriguez et al. 2019)

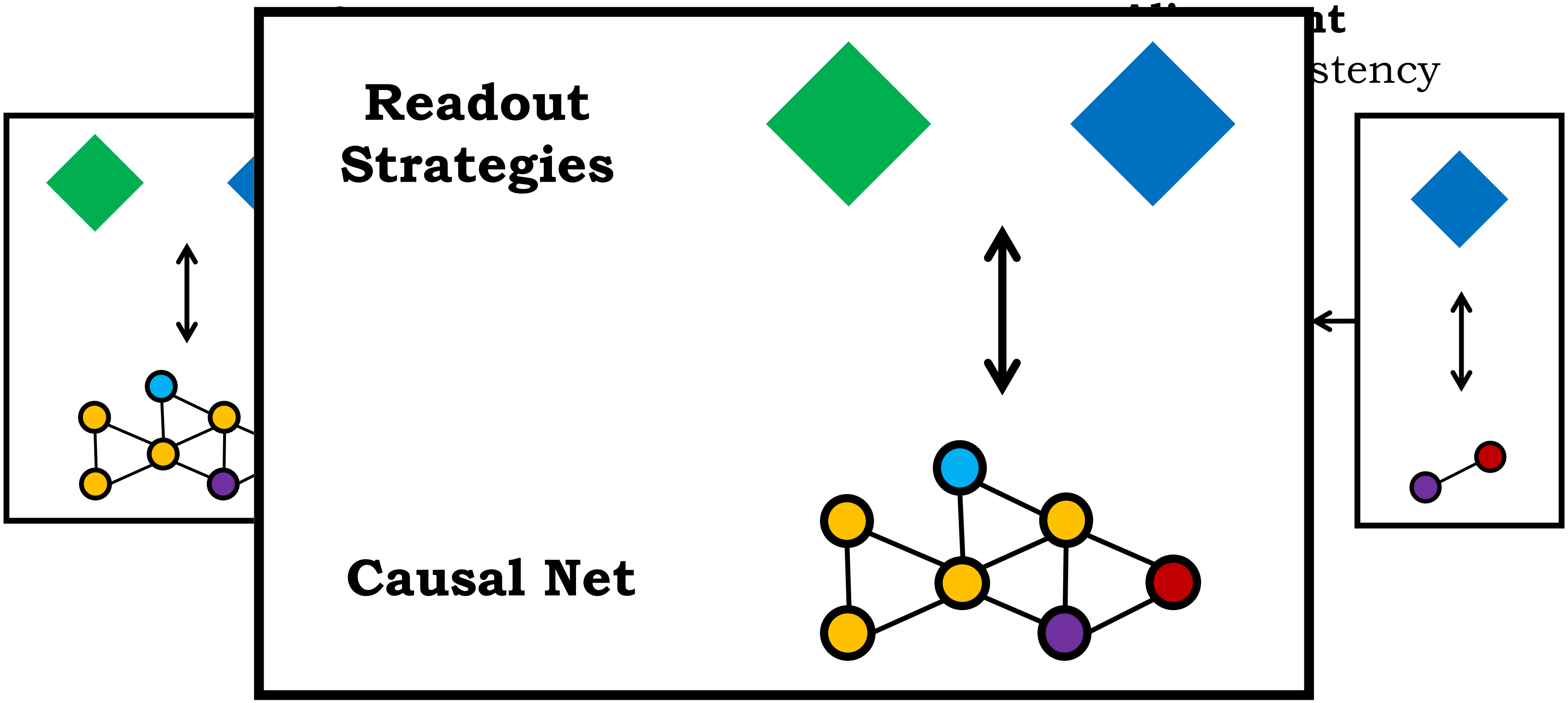
How are we conceptualizing how students make sense of graphs?

Coordination class theory describes a *concept* as a system of strategies for gaining information and knowledge elements that can be used to turn information readouts into inferences



diSessa & Sherin, 1998
Wittman, 2002; Parnafes, 2007; Balabanoff et al, 2020

Span and alignment describe the sophistication of a coordination class



diSessa & Sherin, 1998

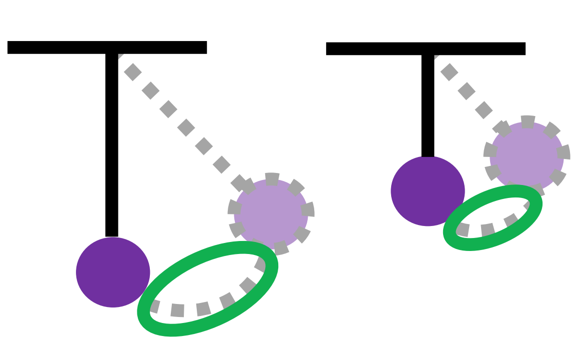
Example of low alignment: How does length influence movement?

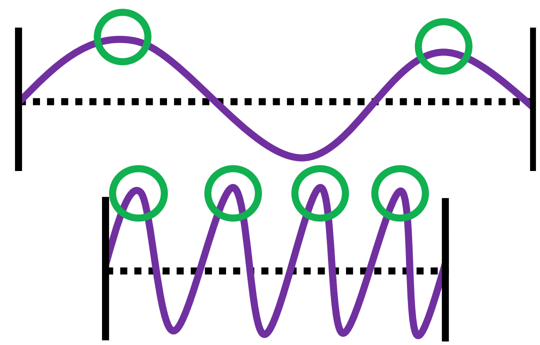
Context

Pendulum

String

Readout





Attending to
“more distance”

Attending to
“more peaks”

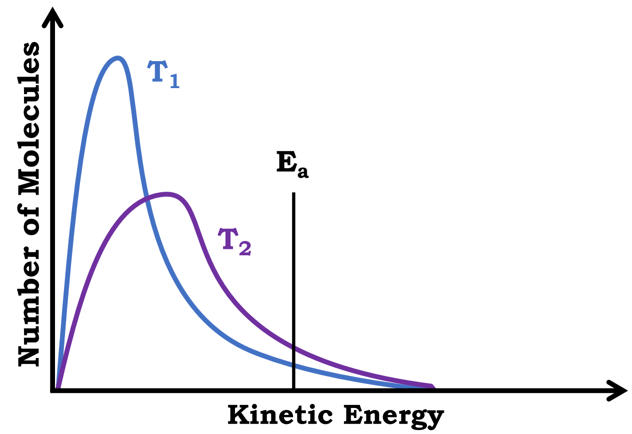
Inference

“Longer is faster”

“Shorter is faster”

Parnafes (2007)

In what ways do students interpret and make inferences from probability distribution graphs?



Is the reaction at T_1 or the reaction at T_2 faster? Why?

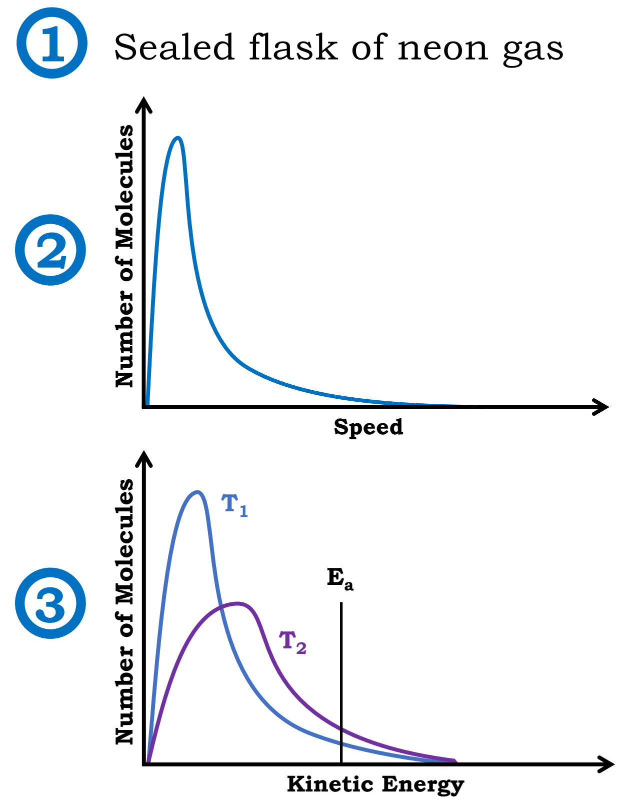
We conducted semi-structured interviews that addressed students' interpretation and use of graphical representations

Sample

N=12
STEM majors
General chemistry II

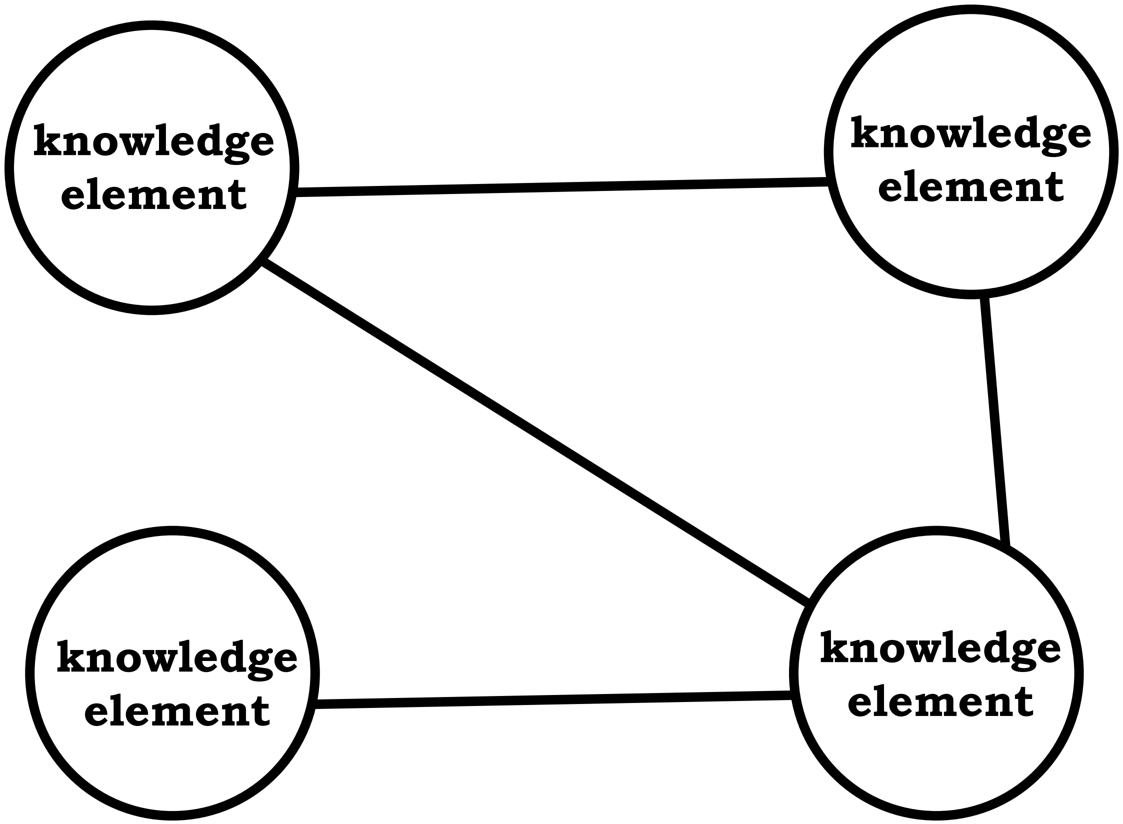
Analysis

Transcribed/processed data
Open-coded
Constructed resource graphs



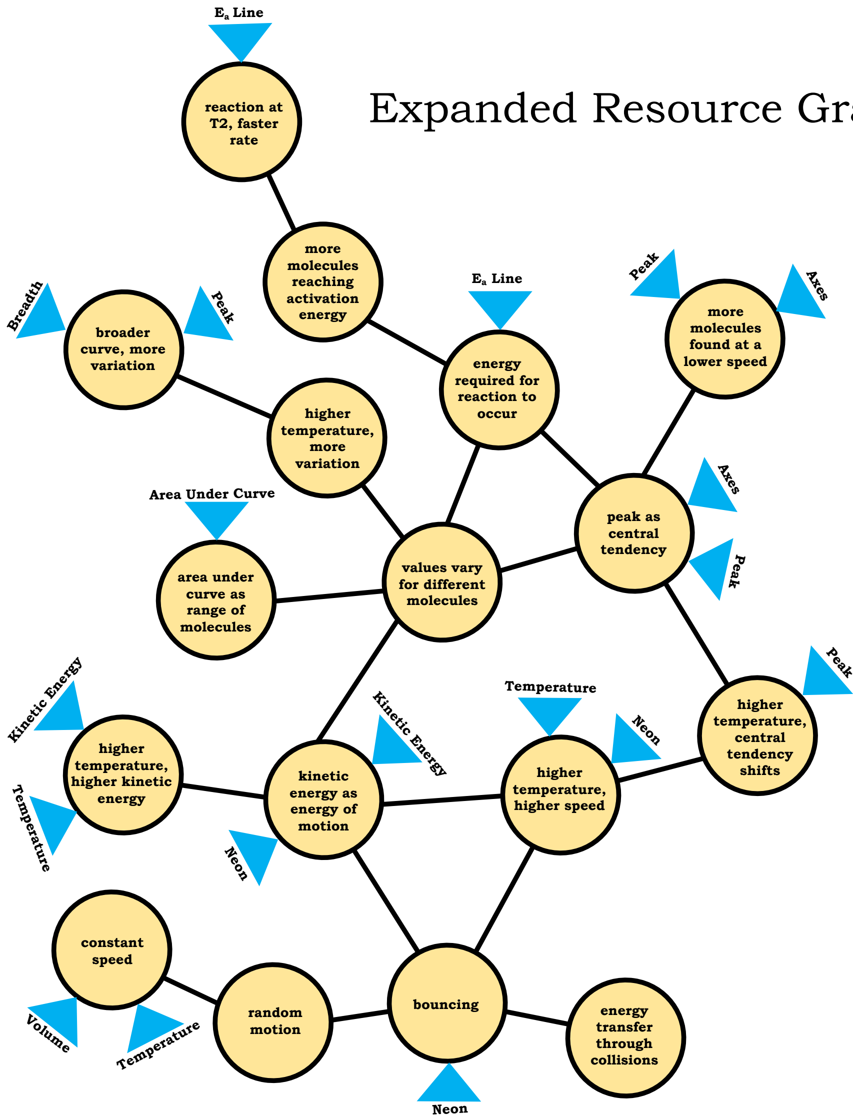
Marton, 1981, 1986; Strauss & Corbin, 1990

Resource graphs indicate the relationship between students' ideas

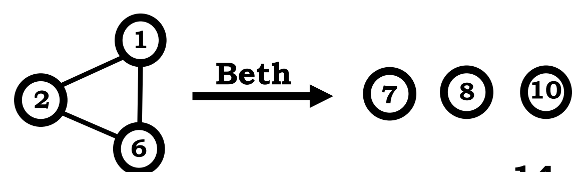


Wittmann, 2006; Sayre & Wittmann, 2008

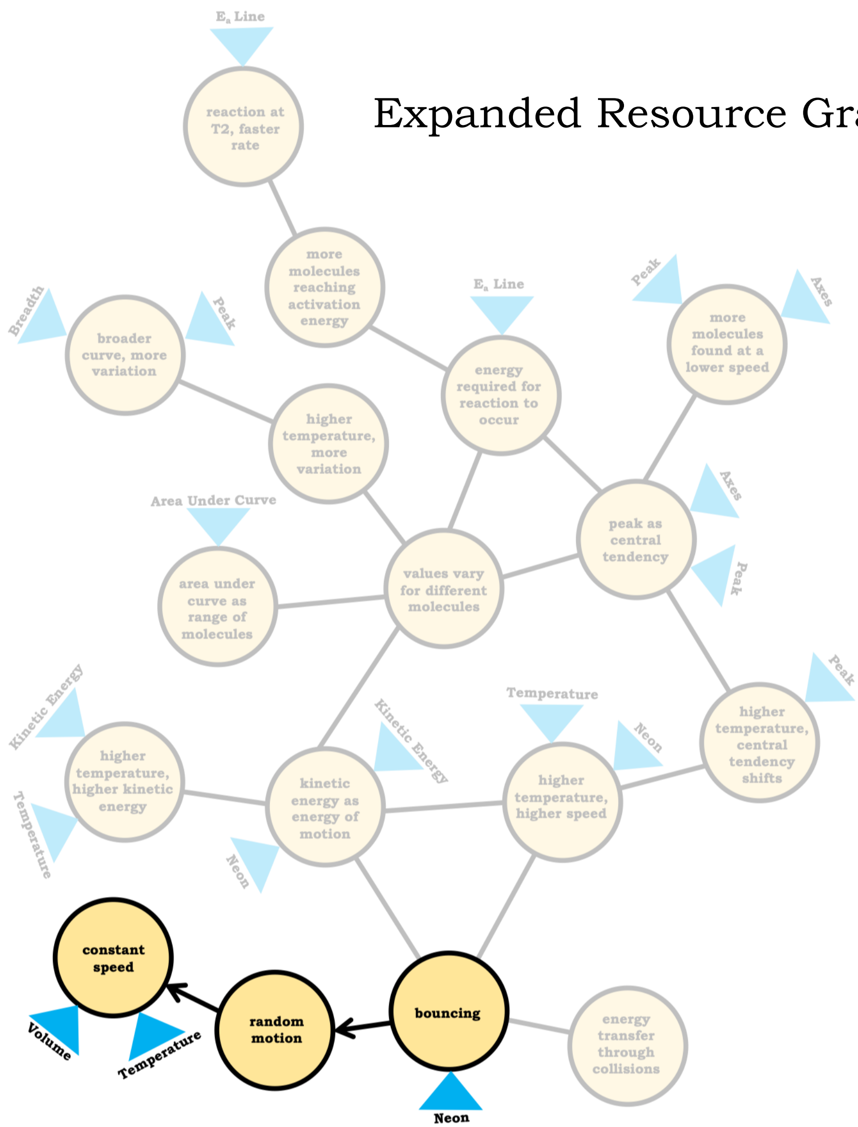
Expanded Resource Graph



Condensed Resource Graph



Expanded Resource Graph



Interviewer: *"So, for our first scenario, I'd like you to imagine that you've got a flask with neon gas inside and that it's in a sealed container, constant volume, constant temperature, at, say, room temperature. So, in general, what would you imagine neon particles to be doing?"*

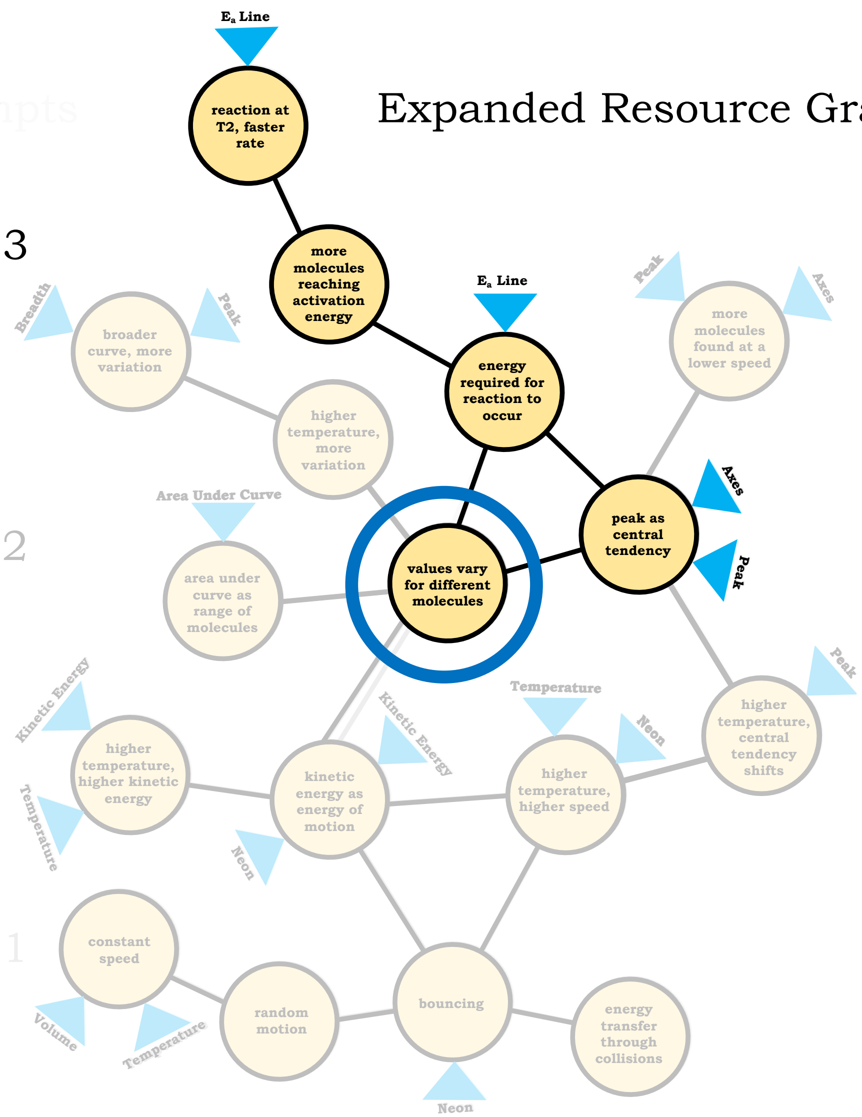
Beth: *"… I would, I guess picture the* ***neon particles****, one little particle,* ***bouncing*** *around in the glass and the flask bouncing out and colliding with other ones and moving quickly, not staying in a step position, but rather having like* ***random motion****. And then not really, I would say* ***not changing speed*** *or anything since the* ***volume*** *and* ***temperature*** *are constant. So, it'd be a pretty like, the same speed that it was moving around."*

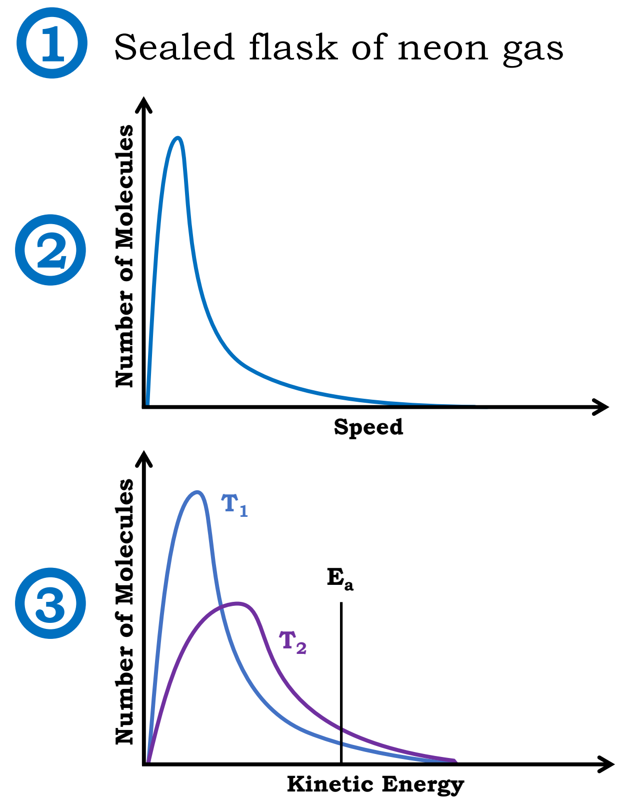
All Prompts   Expanded Resource Graph

Prompt 3

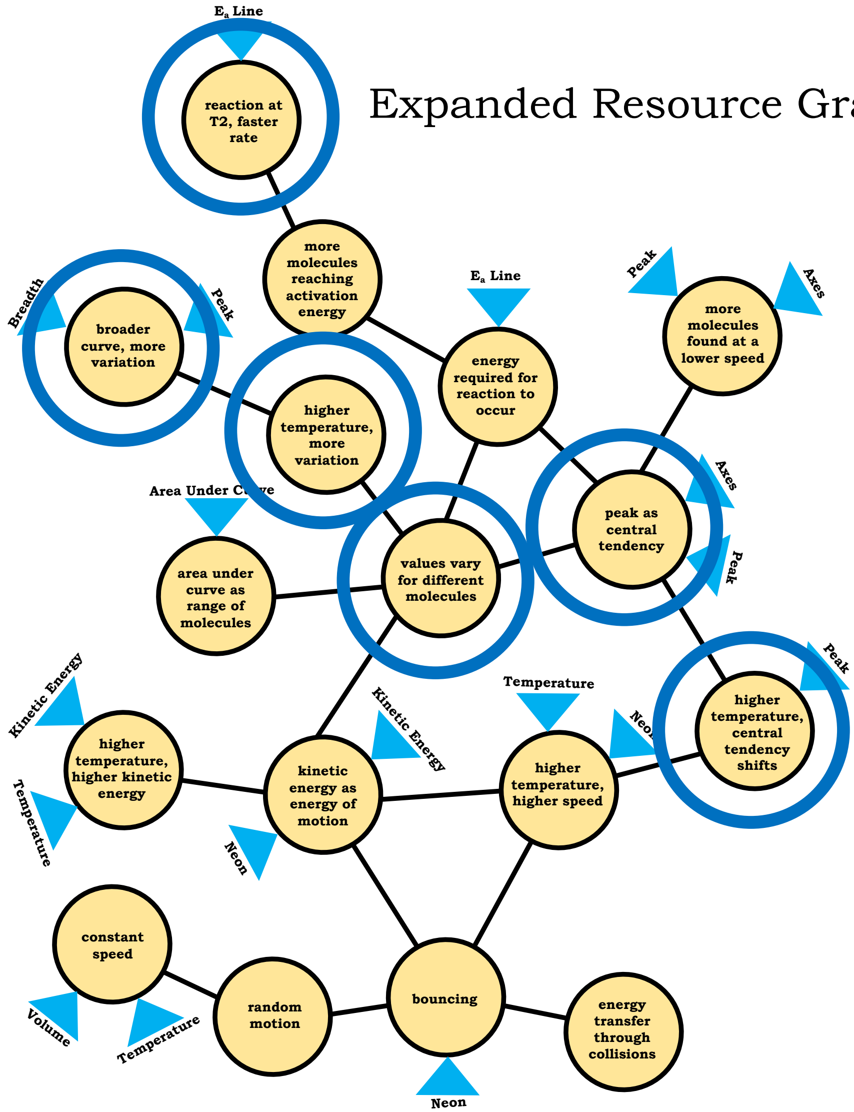
Prompt 2

Prompt 1



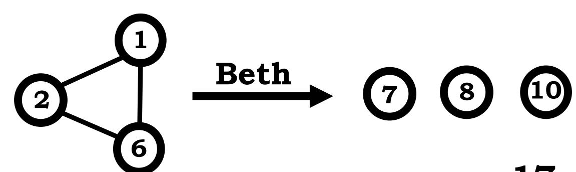


Expanded Resource Graph

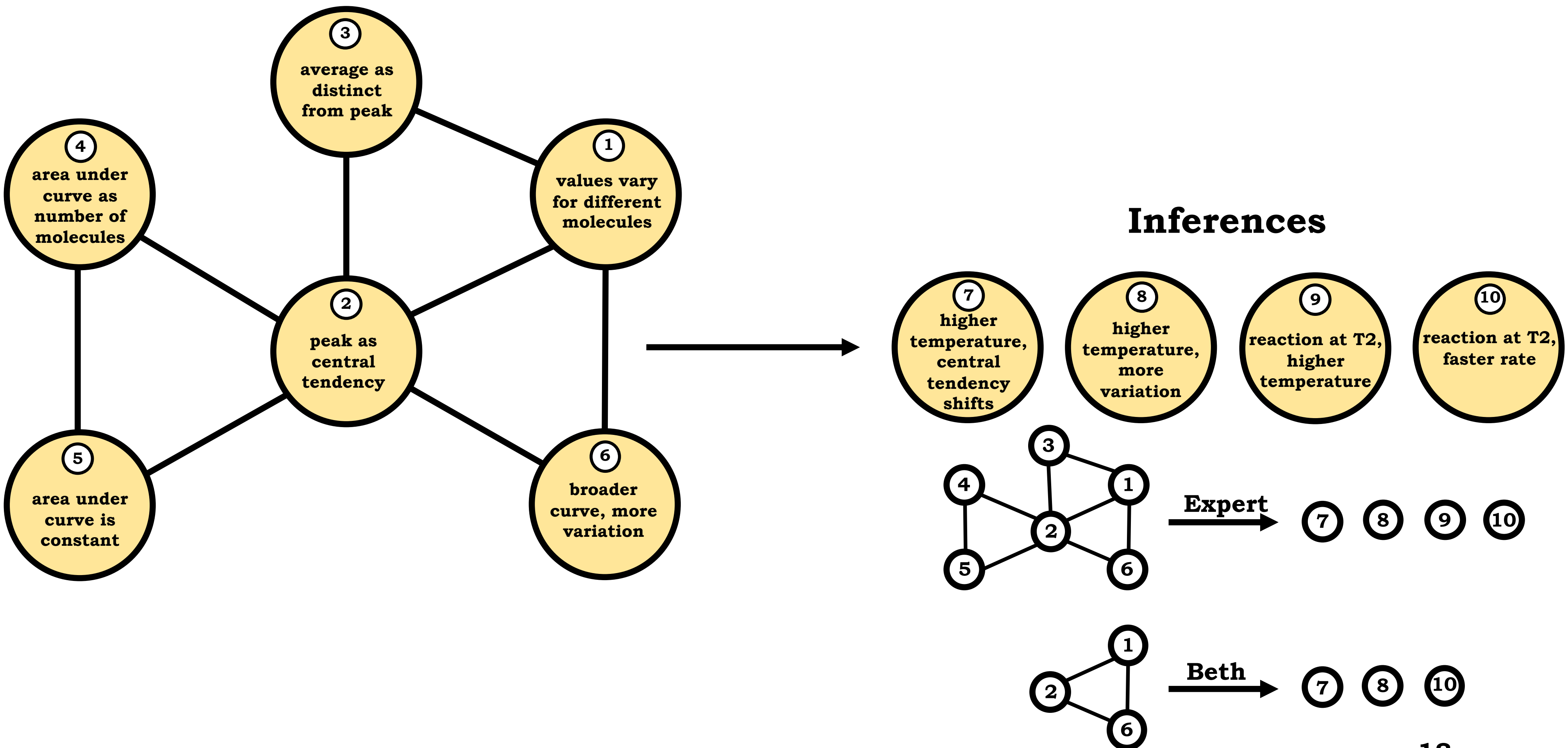


Knowledge elements related to the varied population schema

Condensed Resource Graph

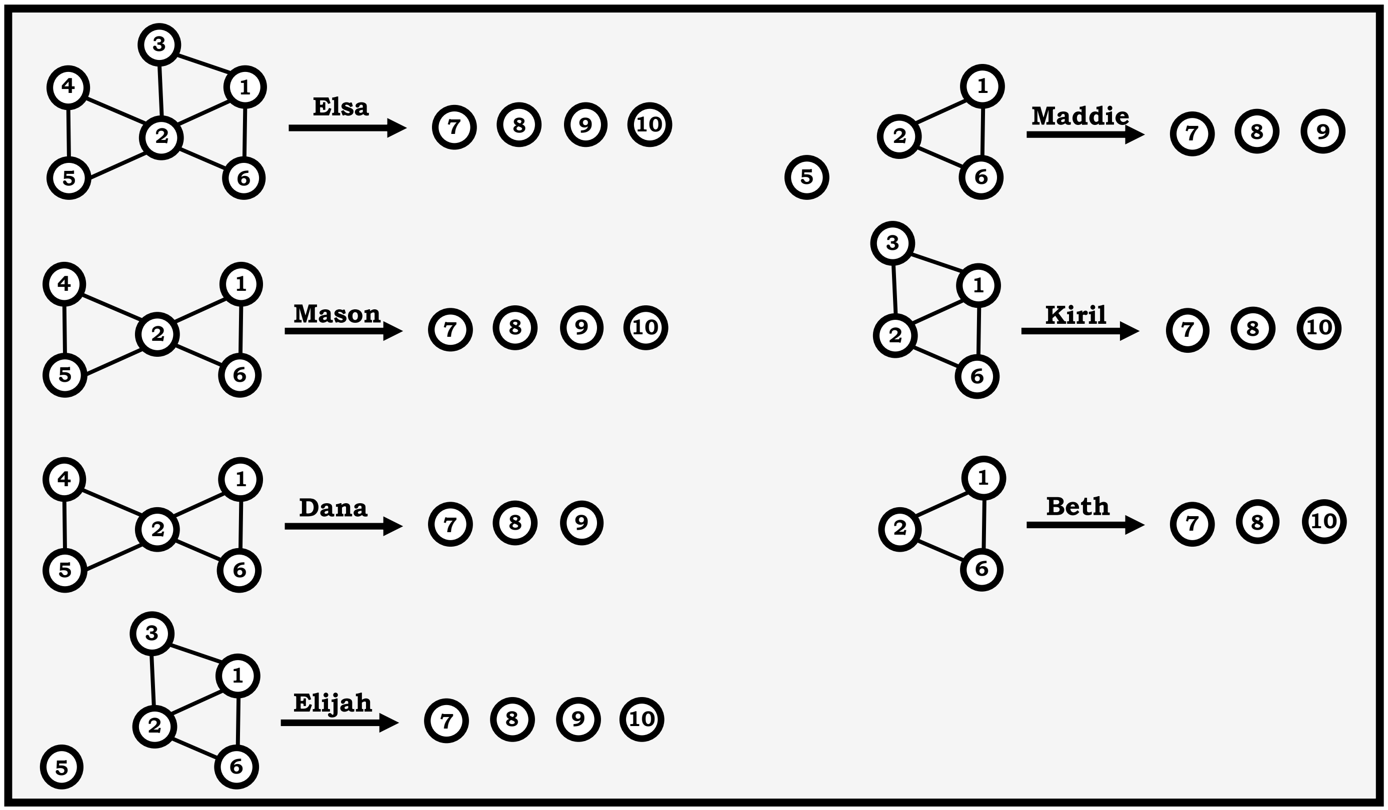


Condensed Resource Graph

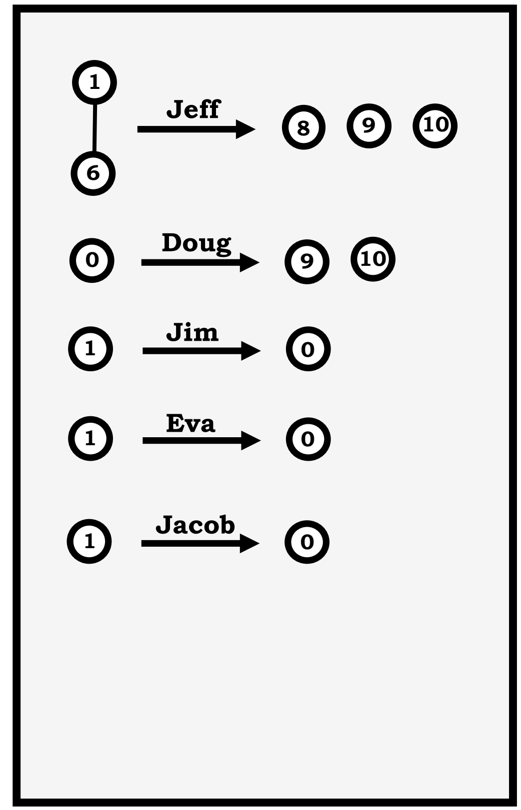


Analysis yielded two distinct readout strategies

Graph as a Distribution (n = 7)

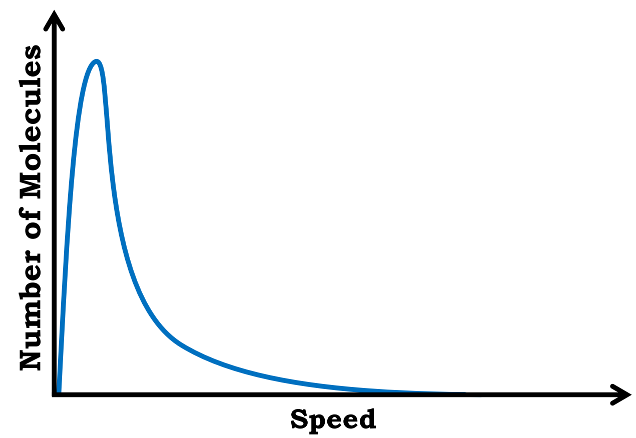


Graph as a Process (n = 5)

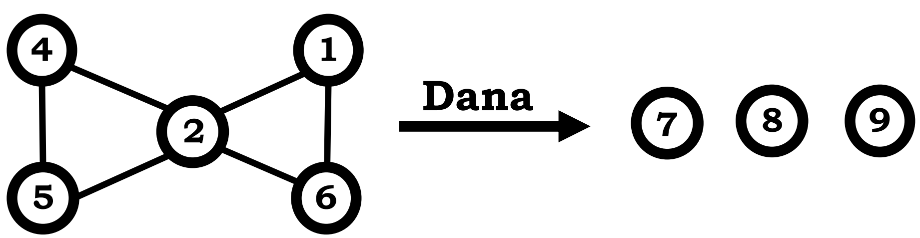


Dana viewed the graph as a distribution

Dana: "And so then as far as the relationship between your axes, the area under the curve was the total number of molecules that are involved and it's more, **kind of like a histogram** almost. And that, the number of molecules associates to **how many molecules are moving at that given speed** at a given time."

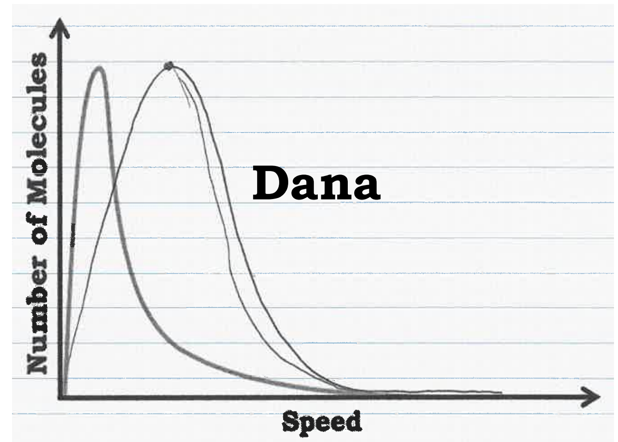


How would you describe the graph?

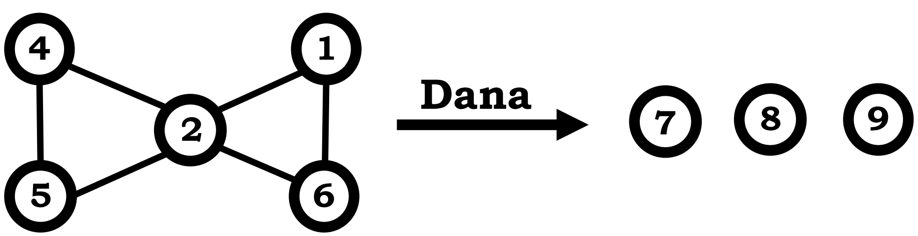


Dana viewed the graph as a distribution

Dana: "But if this is at a higher temperature, then **the maximum will move over** here [shifted peak] 'cause more molecules will be at a higher speed."

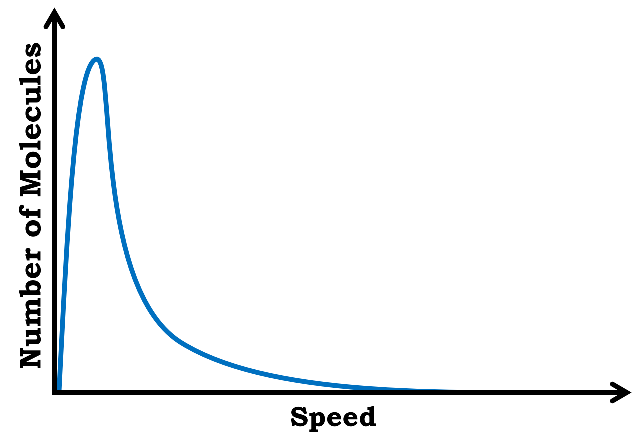


How would the graph change at a higher temperature?



Eva viewed the graph as mapping to a process

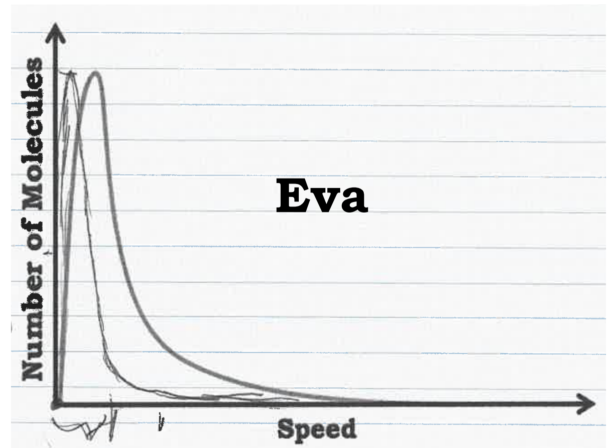
Eva: "So, it looks like what's happening is maybe as the kinetic energy goes up, there are collisions, and more collisions and more reactions happening. And maybe neon is becoming like an elemental molecule. So, **it's going from like a single neon to two neons bound together**. I can't remember if neon works that way, but that's the only, that's the best explanation I have. ... because **the number of molecules decreases as the speed goes up**."



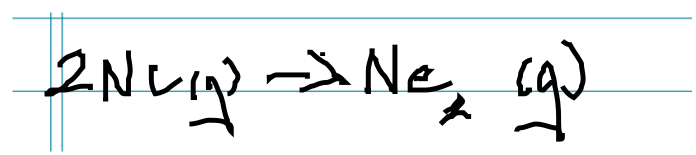
How would you describe the graph?

Eva viewed the graph as mapping to a process

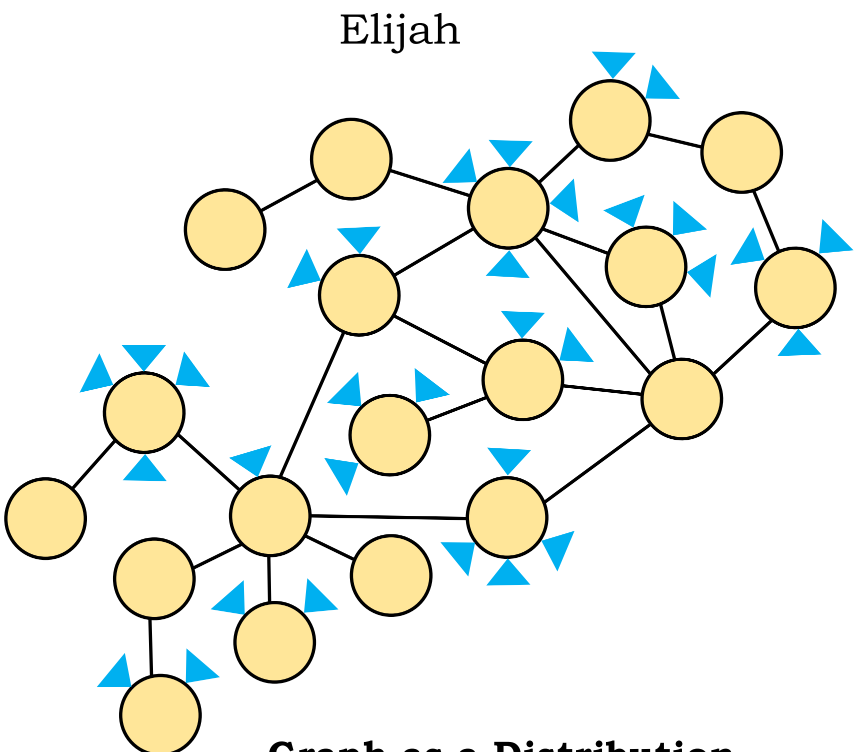
Eva: "So, my thinking was that the **graph would be really compressed, that it would, reactions would start occurring a lot sooner**. I don't know if they'd start disassociating again, but this is what I'm confident, what I'm reasonably confident would happen. … I just figured if **increased kinetic energy caused the number of molecules to decline a lot faster**. Then temperature, since temperature increases, kinetic energy would just expediate [sic] that effect."



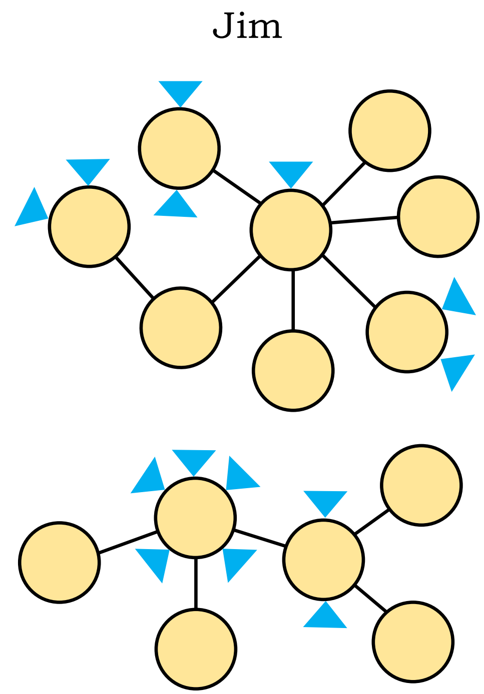
How would the graph change at a higher temperature?



Viewing the graph as a process resulted in a limited span for the varied population coordination class



Graph as a Distribution

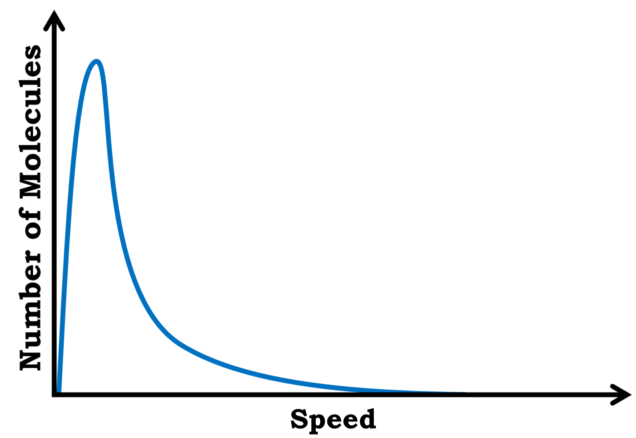


Graph as a Process

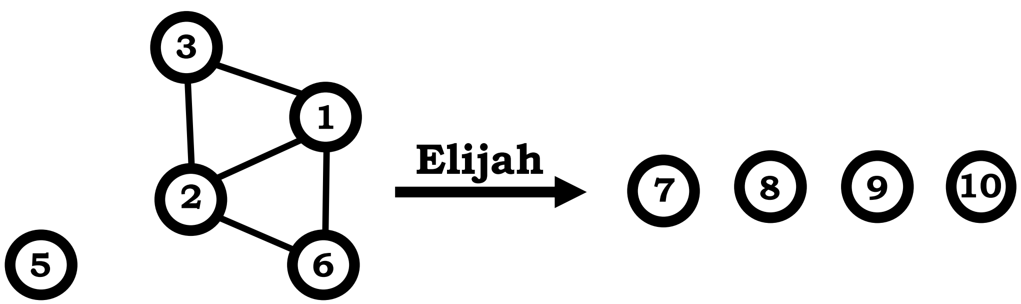
Viewing the graph as a distribution resulted in a broader span for the varied population coordination class

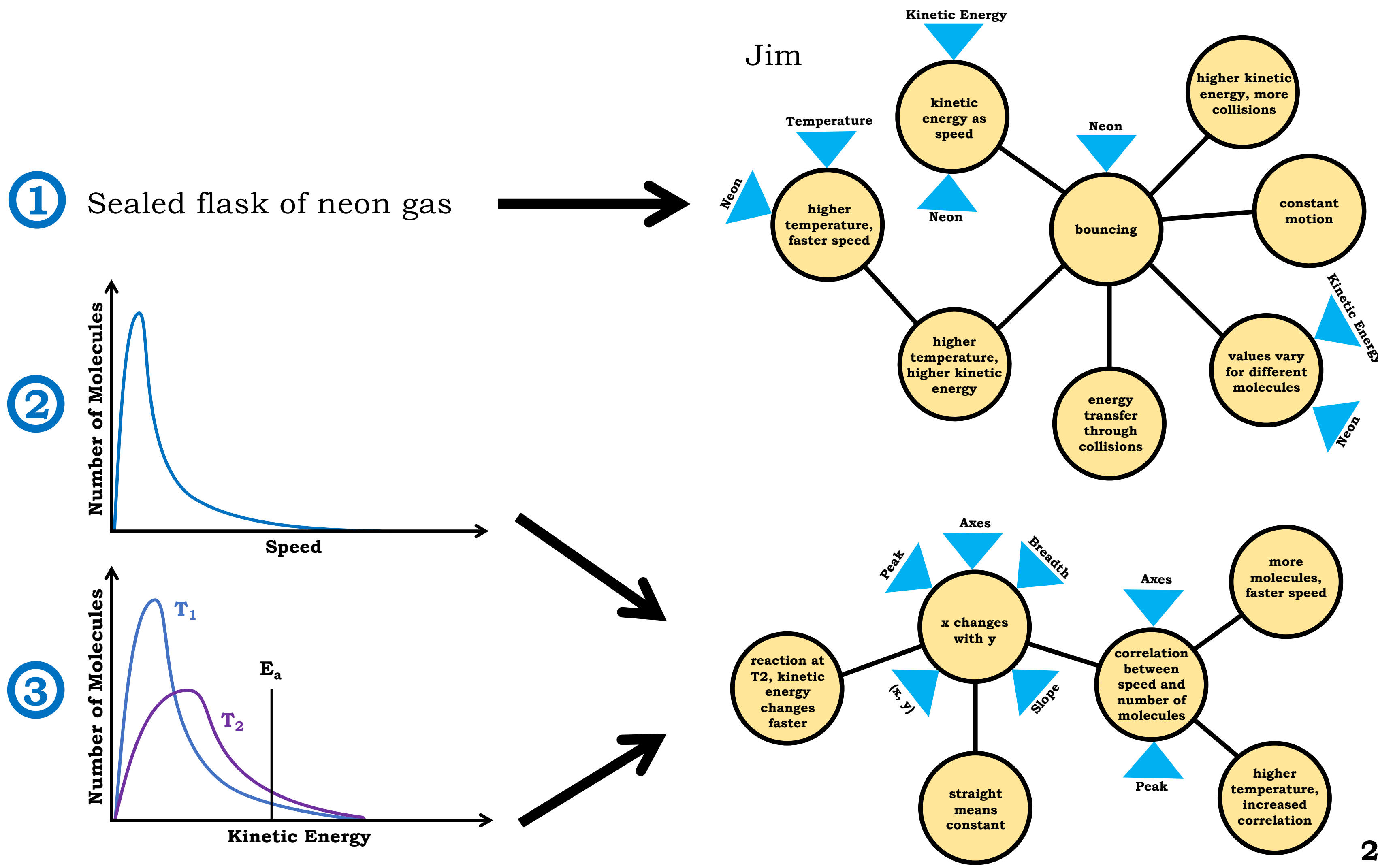
Graph as a Distribution

Elijah: "… **the highest peak, it's at that value of speed that we're getting the most molecules in the neon vessel**. … we have the highest number of molecules moving at a specific speed, the average is likely to be close to that. … it would be fairly close towards the right on the peak. … Yeah, curves down [tail end of the graph]. But I feel like that has more effect on the graph there by **shifting the actual average speed** of the molecules towards the right."



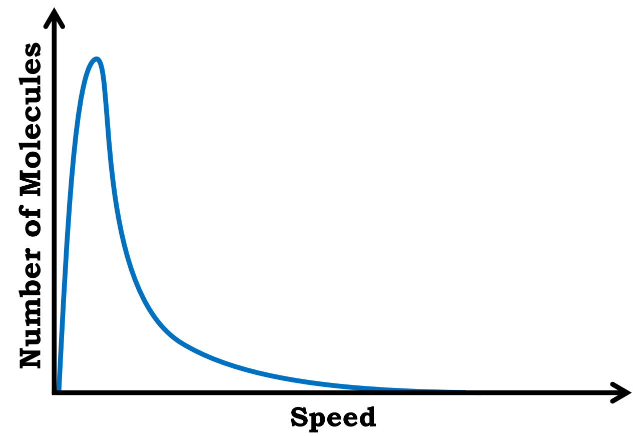
How would you describe the graph?




1
Sealed flask of neon gas
2
Number of Molecules
Speed
3
Number of Molecules
Kinetic Energy
T1
T2
Ea
Jim
Kinetic Energy
kinetic energy as speed
Neon
Temperature
higher temperature, faster speed
Neon
Neon
bouncing
higher kinetic energy, more collisions
constant motion
higher temperature, higher kinetic energy
energy transfer through collisions
values vary for different molecules
Kinetic Energy
Neon
Axes
Peak
Breadth
x changes with y
(x, y)
Slope
reaction at T2, kinetic energy changes faster
straight means constant
Axes
correlation between speed and number of molecules
Peak
more molecules, faster speed
higher temperature, increased correlation

Viewing the graph as a process resulted in a limited span for the varied population coordination class

Jim: "If number of molecules go **higher**, the speed would **decrease**, and if number of molecules go **lower**, the speed would **increase**."

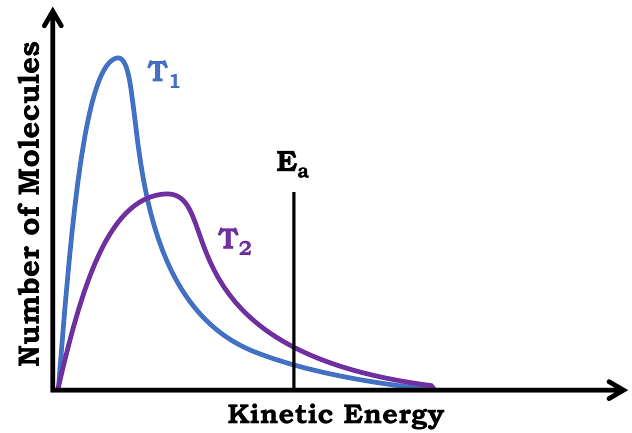


How would you describe the graph?

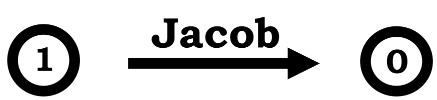
Viewing the graph as a process resulted in limited alignment across inferences

Graph as a Process

Jacob: "**T2 would give you the higher rate of reaction**. … Since it, this original increase in the **number of molecules isn't as high [peak height]** as the T1 graph."



Is the reaction at T_1 or the reaction at T_2 faster?



Some takeaways

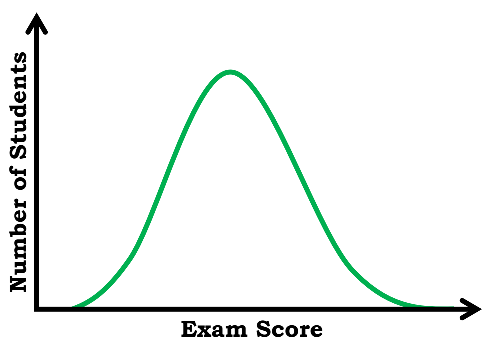
- Although students recognized that variation exists in a system in general terms (e.g., different molecules move at different speeds), they tended to not connect this idea to distribution graphs

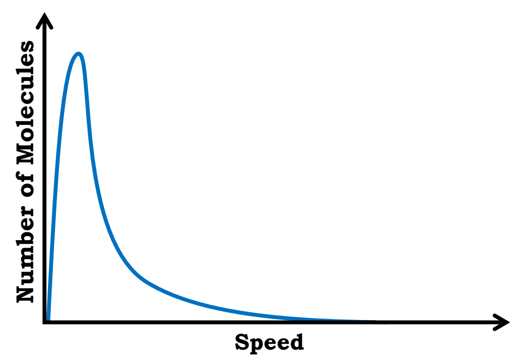
- When provided with frequency distribution graphs such as number of molecules vs. speed, the students tended to interpret the graphs in a way that is analogous to graphs typically presented in chemistry (e.g., “as speed increases, the number of molecules increases”)

- Students need more support recognizing that distribution graphs are intended to be interpreted differently than other graphs (i.e., no causal relationship between x and y) and students need more support explicitly connecting this representation to the variation observed in a system

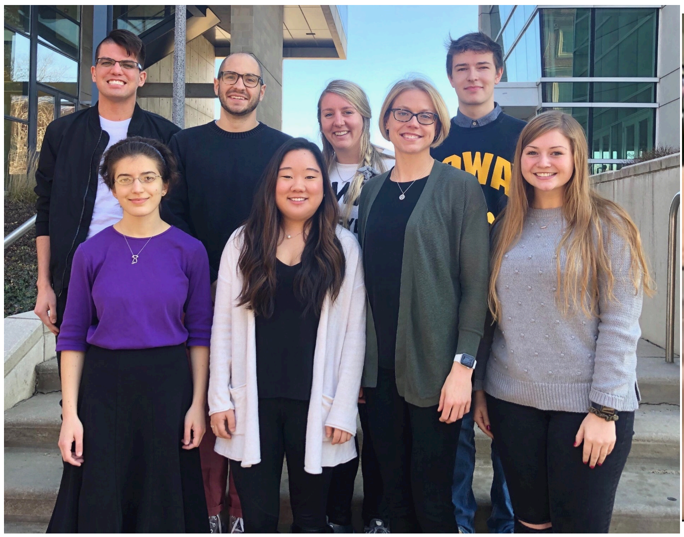
How can instructors support students?

- Even students who seemed to have a fragmented causal net related to the varied population schema still had productive ideas that could potentially be leveraged by instruction
- We posit that the key knowledge element for increasing span is the idea of peak as central tendency
- The use of examples of distribution graphs that students are familiar with may help support students' reasoning with the varied population schema





Thank You



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